

Estimation of multi-mode coupled galloping of multi-span ice-accreted transmission conductors

Yanchi Wu¹, Xinzhong Chen²

¹ School of Highway, Chang'an University, Xi'an, China. Email: yanchi.wu@chd.edu.cn

² National Wind Institute, Department of Civil, Environmental and Construction Engineering, Texas Tech University, Lubbock, USA. Email: xinzhong.chen@ttu.edu

SUMMARY:

This study presents a nonlinear analysis framework of galloping of multi-span ice-accreted transmission conductors suspended by insulators. The aerodynamic forces on the conductor are modelled using quasi-steady theory. The linear equations of motion are also derived with respect to the static equilibrium. The proposed analysis framework is applied to predict the galloping of a two-span four-bundled ice-accreted transmission conductor through complex eigenvalue analysis and response history analysis. A harmonic balance approach is also proposed for a direct calculation of steady-state galloping response. The galloping characteristics in term of initiation condition and coupled steady-state response are investigated. The longitudinal motions of insulators result in reduced vertical modal frequencies of conductor, thus affect the characters of three-dimensional galloping.

Keywords: galloping, multi-span conductor, steady-state amplitude

1. INTRODUCTION

The galloping of an ice-accreted transmission conductor of single span with hinged supports have been extensively studied in literature (e.g., Chen and Wu, 2021). Multi-span conductors are also sensible to galloping and show different characters where the conductor vibrations of adjacent spans and vibration of insulators are coupled (Wang and Lilien, 1998). The quantification of galloping of multi-span conductors with finite element modelling (FEM) and response history analysis (RHA) is computationally inefficient. This study presents an efficient nonlinear analysis framework and characterization of galloping of multi-span ice-accreted transmission conductors using equations of modal displacements.

2. THEORETICAL BACKGROUND

A horizontal multi-span transmission conductor supported by insulators which are pin suspended from towers and modeled as pendulums is considered (Fig.1). The influence of tower vibration on galloping of conductor is considered to be negligible (Wang and Lilien, 1998). The wind loads per unit length acting on the i -th span conductor in vertical (y), horizontal (z) and torsional (θ) directions are denoted as $f_{vi}(x, t)$, $f_{wi}(x, t)$ and $f_{\theta i}(x, t)$, which are determined based on quasi-steady theory. The conductor displacements relative to insulators are $v_i(x, t)$, $w_i(x, t)$ and $\theta_i(x, t)$. The end displacements of the i -th insulator are $u_{si}(t)$, $v_{si}(t)$ and $w_{si}(t)$, respectively. The displacements of the conductor relative to insulators are expressed in modal displacements as:

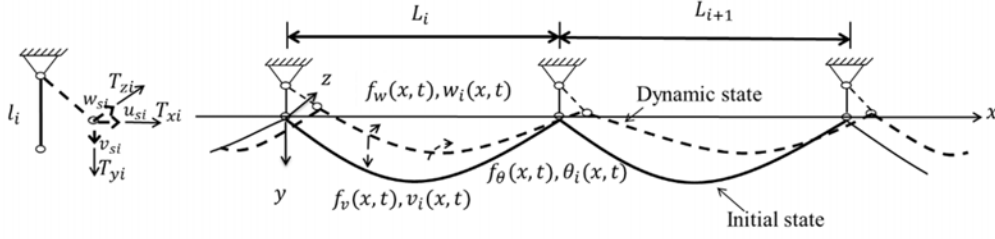


Fig. 1 Deformation of the multi-span transmission conductor

$$v_i(x, t) = \sum_{j=1}^{N_v} \varphi_{ijv}(x) q_{ijv}(t); \quad w_i(x, t) = \sum_{j=1}^{N_w} \varphi_{ijw}(x) q_{ijw}(t); \quad \theta_i(x, t) = \sum_{j=1}^{N_\theta} \varphi_{ij\theta}(x) q_{ij\theta}(t) \quad (1)$$

where $\varphi_{ijv}(x)$, $\varphi_{ijw}(x)$ and $\varphi_{ij\theta}(x)$ are j -th in-plane, out-of-plane and torsional mode shapes of the i -th conductor when both ends are hinged; $q_{ijv}(t)$, $q_{ijw}(t)$ and $q_{ij\theta}(t)$ are the modal displacements; N_v , N_w and N_θ are the mode numbers considered.

The equations of motion of the i -th span conductor are

$$M_{ijvL} \ddot{v}_{si} + M_{ijvR} \ddot{v}_{s(i+1)} + M_{ijv} (\ddot{q}_{ijv} + 2\xi_{ijv} \omega_{ijv} \dot{q}_{ijv}) + (H_i + \Delta H_i) \sum_{k=1}^{N_v} a_{ijkv} q_{ikv} + a_{0ijv} \Delta H_i = Q_{ijv} \quad (j = 1, 2, \dots, N_v) \quad (2)$$

$$M_{ijwL} \ddot{w}_{si} + M_{ijwR} \ddot{w}_{s(i+1)} + M_{ijw} \left(\ddot{q}_{ijw} + 2\xi_{ijw} \omega_{ijw} \dot{q}_{ijw} + \omega_{ijw}^2 \left(1 + \frac{\Delta H_i}{H_i} \right) q_{ijw} \right) = Q_{ijw} \quad (j = 1, 2, \dots, N_w) \quad (3)$$

$$M_{ij\theta} (\ddot{q}_{ij\theta} + 2\xi_{ij\theta} \omega_{ij\theta} \dot{q}_{ij\theta} + \omega_{ij\theta}^2 q_{ij\theta}) = Q_{ij\theta} \quad (j = 1, 2, \dots, N_\theta) \quad (4)$$

and the dynamic tension ΔH_i is calculated as

$$\frac{\Delta H_i L_i}{EA_i} = \Delta u_{si} + \frac{1}{L_i} \Delta v_{sio} \Delta v_{si} + \sum_{j=1}^{N_v} a_{0ijv} q_{ijv} + \frac{1}{2} \sum_{j=1}^{N_v} \sum_{k=1}^{N_v} a_{ijkv} q_{ijv} q_{ikv}(t) + \frac{1}{2} \sum_{j=1}^{N_w} a_{ijw} q_{ijw}^2 \quad (5)$$

where M_{ijv} , M_{ijw} and $M_{ij\theta}$, ω_{ijv} , ω_{ijw} and $\omega_{ij\theta}$, ξ_{ijv} , ξ_{ijw} and $\xi_{ij\theta}$ are generalized mass, modal frequencies and damping ratios of the i -th span conductor with hinged supports at both ends; L_i and H_i , are span length and initial horizontal tension of the i -th span conductor.

For the i -th insulator, the end forces and displacements are related as follows

$$T_{xi} = (H_i + \Delta H_i) - (H_{i-1} + \Delta H_{i-1}) \quad (6)$$

$$T_{yi} = (H_i + \Delta H_i) \left(\frac{m_i g L_i}{2H_i} + v_i'(0, t) \right) - (H_{i-1} + \Delta H_{i-1}) \left(-\frac{m_{i-1} g L_{i-1}}{2H_{i-1}} + v_{i-1}'(L_{i-1}, t) \right) \quad (7)$$

$$T_{zi} = (H_i + \Delta H_i) w_i'(0, t) - (H_{i-1} + \Delta H_{i-1}) w_{i-1}'(L_{i-1}, t) \quad (8)$$

$$u_{sio} + u_{si} = \frac{T_{xi}}{T_i} l_i; \quad v_{sio} + v_{si} = \left(\frac{T_{yi}}{T_i} - 1 \right) l_i; \quad w_{sio} + w_{si} = \frac{T_{zi}}{T_i} l_i \quad (9)$$

where $T_i = \sqrt{T_{xi}^2 + T_{yi}^2 + T_{zi}^2}$, u_{sio} , v_{sio} , and w_{sio} are static end displacements of the i -th insulator due to static load.

It is evident that the vertical (in-plane) and lateral (out-of-plane) displacements of insulator and the displacements of two adjacent span conductors are coupled and have nonlinear relations. The equations of motions can be solved by using step-by-step integration method, where the insulator displacements are represented by the displacements of conductors through static condensation. The Harmonic Balance (HB) method can also be applied to obtain approximate periodic steady-state response. The linear equations of motion around the static equilibrium can be derived when the vibration amplitude is small. The modal analysis of these linear equations of motion with linearized aerodynamic forces provides the initiation conditions of galloping.

3. GALLOPING CHARACTERISTICS OF A TWO-SPAN BUNDLED CONDUCTOR

A semi-suspension ice-accreted four-bundled transmission conductor of equal span length (244 m) with both hinged supports and connected by an insulator (2 m) at a same elevation is considered. The fundamental symmetric modal frequencies of in-plane, out-of-plane, and torsional vibrations of a pin-supported single span conductor are 0.421 Hz, 0.267 Hz and 0.377 Hz, respectively. The modal damping ratios are assumed to be 0.5%. The cross-section is shown in Fig.2. The ice accreted cross-sectional model has a sharp triangular tip on each conductor. The static force coefficients reported in Matsumiya et al (2018) are used (Fig.3).

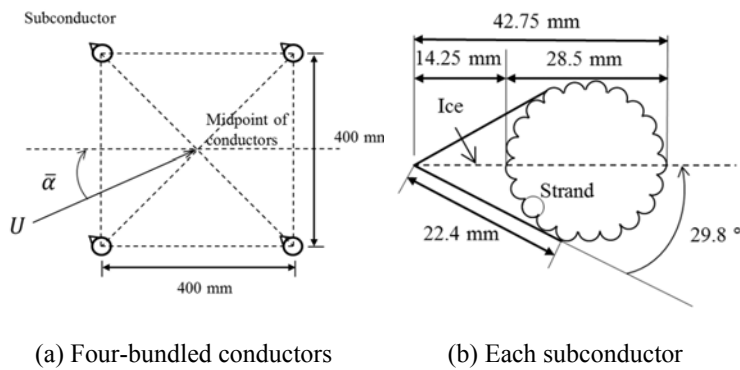


Fig. 2 Cross-section of the conductor

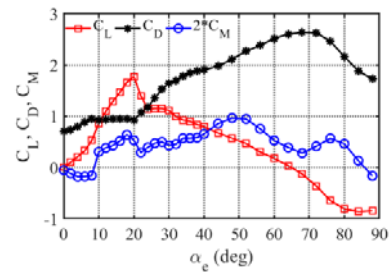
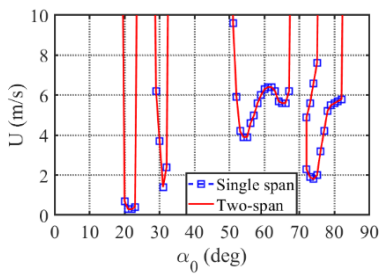


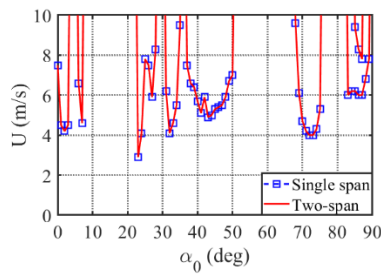
Fig. 3 C_L, C_D and C_M

The complex eigenvalue analysis using linear equations of motion and linear aerodynamic force modelling determines the initiation condition of multi-mode coupled galloping. Fig. 4 shows the onset wind speed of vertical and torsional galloping as a function of ice location angle α_0 with zero angle of attack. The symmetric (in-phase) vertical and torsional galloping regions of the two-span conductor are the same as those of single span conductor. On the other hand, the anti-symmetric (out-of-phase) vertical and torsional galloping regions are different from those of single span conductor, due to the reduced frequency of vertical anti-symmetric mode (V-A-1).

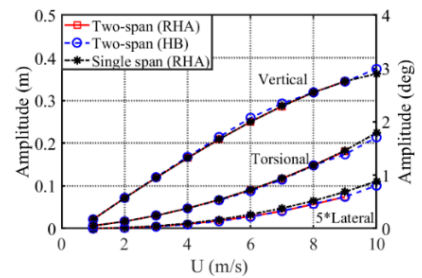
Fig. 5 shows the steady-state amplitudes of displacements at the first span center for vertical symmetric (V-S-1) and anti-symmetric (V-A-1) mode branch galloping at different wind speeds calculated from RHA with $\alpha_0 = 20\text{deg}$. Fig. 6 displays an example of galloping motion. The results of single span are also compared. The estimations of HB method are also presented and match the results very well. The calculation based on the proposed analytical framework is computationally very effective and accurate as compared to the FEM approach.



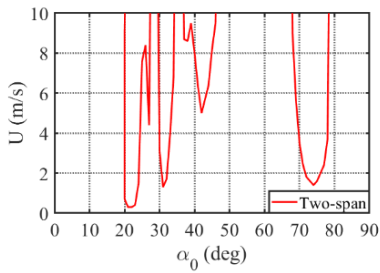
(a) Vertical in-phase



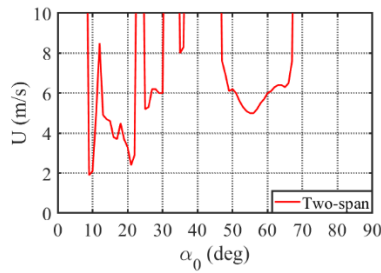
(b) Torsional in-phase



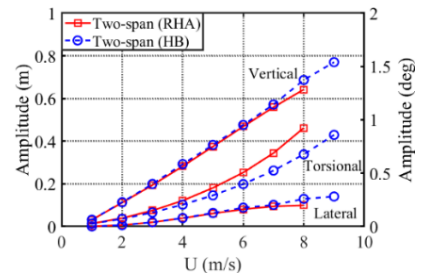
(a) V-S-1 galloping



(c) Vertical out-of-phase



(d) Torsional out-of-phase



(b) V-A-1 galloping

Fig. 4 Onset wind speed of galloping

Fig.5 Galloping amplitudes ($\alpha_0=20$ deg)

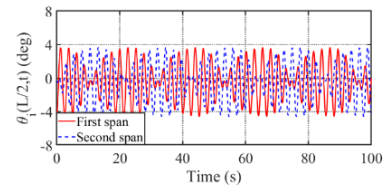
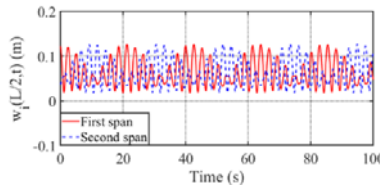
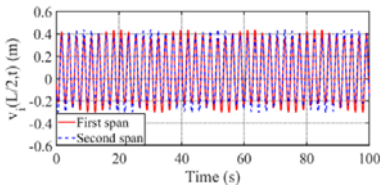


Fig. 6 Time histories of 3D displacements at two span centers ($\alpha_0 = 20$ deg, $U = 10$ m/s)

4. CONCLUSION

An efficient analysis approach of multi-span conductor galloping was developed and verified. The galloping of multi-span conductors shows different characters from single span conductors due to the coupling effect of adjacent spans and insulators. The harmonic balance approach was also developed for predicting steady-state amplitudes of galloping motion and its accuracy was confirmed.

REFERENCES

- Chen, X. and Wu, Y., 2021. Explicit closed-form solutions of the initiation conditions for 3DOF galloping or flutter. *J. Wind Eng. Ind. Aerodyn.*, 219, 104787.
- Matsumiya, H., Nishihara T. and Yagi, T., 2018. Aerodynamic modeling for large-amplitude galloping of four-bundled conductors, *J. Fluids Struct.* 82, 559-576.
- Wang, J., and Lilien, J.-L., 1998. Overhead electrical transmission line galloping: A full multi-span 3-DOF model, some applications and design recommendations. *IEEE Trans. Power Deliv.*, 13(3), 909-916.